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Two major papers were written during the last year, describing our work under this contract in our two main areas of research: (a) numerically induced chaos, and (b) waves in shallow water. This progress report summarizes the contents of those two papers.

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Nonlinear Ocean Waves

Progress Report: 1/1/93-12/31/93

The research conducted under this contract splits naturally into two main topics: (a) numerically induced chaos; and (b) waves in shallow water. During 1993, we wrote major papers on each of these topics. This report summarizes the contents of these two papers.

(a) Numerically induced chaos

The cubic-nonlinear Schrödinger equation in one dimension,

$$i\partial_t\psi + \partial_x{}^2\psi + 2|\psi|^2\psi = 0, \qquad \text{(NLS)}$$

arises in several physical contexts, including the evolution of nearly monochromatic, one-dimensional waves in deep water. The equation is known to be a completely integrable Hamiltonian system, with infinitely many constants of the motion, *etc.* With spatially periodic boundary conditions, one can show that solutions of the equation that are quasi-periodic in time are dense among the set of all solutions, and that the system is demonstrably nonchaotic.

Even so, Ablowitz, Schober, and Herbst (1993) showed that for certain ranges of initial data, discrete numerical approximations of NLS are chaotic, even though the continuous system is nonchaotic. Moreover, they showed that the observed chaotic behaviour occurs for a variety of numerical schemes, even schemes that are themselves completely integrable, and even though these schemes preserve the constants of the motion to a very high accuracy. Even worse, tiny (10^{-16}) roundoff errors grow rapidly, and render numerical simulations meaningless within a few time units.

In other words, even though one can prove that the solution of NLS that evolves from particular initial data is well-behaved (and nonchaotic), numerical approximations of this solution are chaotic, and are completely unreliable after a few time units. We reiterate that this chaotic behaviour appears for every kind of discretization tested. The fact that it appears even for an extremely well-behaved system like NLS raises profound questions about claims of chaotic behaviour in other systems, especially when these claims are based on numerical simulations.

(b) Waves in shallow water

The objective of this work has been to develop a model of waves in shallow water that is both computationally feasible and physically realistic, without resorting to the usual restrictions to waves of infinitesimal amplitude or to waves with one-dimensional surface patterns. The theoretical basis for all of this work is the fact that the Kadomtsev-Petviashvili equation,

$$\partial_x(\partial_t u + u\partial_x u + \partial_x^3 u) + \partial_y^2 u = 0, \qquad \text{(KP)}$$

describes approximately the evolution of waves of moderate amplitude as they propagate primarily in one direction in shallow water of uniform depth. In addition, the equation is known to admit large families of exact periodic and quasi-periodic solutions. The simplest of these ("genus 1") are the cnoidal waves found by Korteweg and deVries a century ago. The next level of complication ("genus 2") includes all KP solutions of permanent form. Typically, these waves of permanent form have two-dimensional surface patterns, and they are characterised by 8 real parameters.

In a paper now under review, Hammack, McCallister, Scheffner, and Segur (1993) demonstrate experimentally the existence of finite-amplitude waves of (nearly) permanent form in shallow water of uniform depth. The surface patterns of these waves are two-dimensional and spatially periodic. The basic template of a wave is hexagonal, but the wave patterns need not be symmetric about the direction of propagation, as required in previous studies. The waves are easy to generate, and they seem to be stable to perturbations. We know of no previous demonstration of genuinely two-dimensional, asymmetric waves of permanent form in shallow water. (In fact, the KP equation also arises in plasma physics and elsewhere. We know of no demonstration of the existence of two-dimensional, periodic waves of permanent form in a plasma, or in any other context in which the KP equation arises. This seems to be the first experimental demonstration of these waves in any physical context.)

In the same paper, the authors also set forth an algorithm to infer the 8 parameters of a KP solution of genus 2 from specific wave-gauge measurements, and they show that the KP solutions so obtained describe the measured waves with reasonable accuracy, even outside the putative range of validity of the KP equation.

References:

M.J. Ablowitz, C. Schober, and B.M. Herbst, "Numerical chaos, roundoff errors, and homoclinic manifolds," *Phys. Rev. Lett.*, 71, pp. 2683-2686, 1993

J. Hammack, D. McCallister, N. Scheffner, and H. Segur, "Two-dimensional periodic waves in shallow water, part 2: asymmetric waves," *J. Fluid Mech, sub judice*, 1993